

Quantifying the effect of intertrial dependence on perceptual decisions

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In the perceptual sciences, experimenters study the causal mechanisms of perceptual systems by probing observers with carefully constructed stimuli. It has long been known, however, that perceptual decisions are not only determined by the stimulus, but also by internal factors. Internal factors could lead to a statistical influence of previous stimuli and responses on the current trial, resulting in *serial dependencies*, which complicate the causal inference between stimulus and response. However, the majority of studies do not take serial dependencies into account, and it has been unclear how strongly they influence perceptual decisions. We hypothesize that one reason for this neglect is that there has been no reliable tool to quantify them and to correct for their effects. Here we develop a statistical method to detect, estimate, and correct for serial dependencies in behavioral data. We show that even trained psychophysical observers suffer from strong history dependence. A substantial fraction of the decision variance on difficult stimuli was independent of the stimulus but dependent on experimental history. We discuss the strong dependence of perceptual decisions on internal factors and its implications for correct data interpretation.

Introduction

In the perceptual sciences, one of the central goals is to infer the causal mechanisms of perceptual systems by probing human observers or animals with carefully constructed or selected stimuli (Blackwell, 1952; Green & Swets, 1966). In practice, a tacit assumption underlying this approach is that the only systematic causal determinant of the perceptual decision is the presented stimulus in an individual trial, i.e., that each response is not influenced by previous responses or stimuli. This would imply that, given the stimulus, different trials of an experiment are statistically independent. While this assumption is undoubtedly convenient, it may, nonetheless, not be appropriate. Internal factors can lead to a statistical influence of previous stimuli and responses on the current trial (Green, 1964). In the current context, internal factors are any factors that may influence an observer's decision other than the stimulus—internal dynamics, attentional and motivational state, adaptation and learning as well as the stimulus or response history in the form of intertrial dependencies.

In principle, the existence of such intertrial dependencies has been recognized long ago (Senders &

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Sowards, 1952; Verplanck, Collier, & Cotton, 1952; Howarth & Bulmer, 1956; Green, 1964), and numerous studies have reported correlations between successive trials in experiments with human observers (Maljkovic & Nakayama, 1994; Mori, 1998; Lages & Treisman, 2010; Bode et al., 2012). Multiple studies have quantified correlations between responses (Howarth & Bulmer, 1956; Mori & Ward, 1995; Mori, 1998; Lages & Treisman, 1998, 2010; Maloney, Dal Martello, Sahm, & Spillmann, 2005) or response times (Peeke & Stone, 1972; Maljkovic & Nakayama, 1994; Wagenmakers, Farrell, & Ratcliff, 2004; Otto & Mamassian, 2012). Some studies have suggested theoretical models for interdependencies between successive trials in particular tasks (Lockhead & King, 1983; Stewart, Brown, & Chater, 2002, 2005; Maljkovic & Martini, 2005; Martini, 2010; Raviv, Ahissar, & Loewenstein, 2012) and tested signatures of model assumptions (Green, Luce, & Duncan, 1977; Baird, Green, & Luce, 1980; Green, Luce, & Smith, 1980; Treisman & Williams, 1984). Although there is some evidence suggesting that the effect of sequential dependencies on human psychometric functions is small (Verplanck & Blough, 1958), studies with animals found that taking intertrial dependencies into account was of critical importance (Lau & Glimcher, 2005; Busse et al., 2011).

This substantial body of literature shows that the existence of nonstimulus effects on perceptual decisions is well established in principle. However, they are commonly ignored in neuroscientific and psychophysical research: We analyzed the 2011 volume of a high-impact neuroscience journal (*Nature Neuroscience*) as well as an established specialist journal for visual psychophysics (*Journal of Vision*) and found that only 1 out of 54 “candidate” articles (Meier, Flister, & Reinagel, 2011) checked and corrected for intertrial dependence (for details, see Appendix A1). The vast majority of articles did not explicitly state that they assume trials to be independent but analyzed their data as if they were. Thus, despite considerable evidence for intertrial dependencies in experiments that were designed to explicitly study them, these results are rarely applied in practice. This is potentially problematic, as nonstimulus determinants could lead to both spurious correlations between behavior and measurements of neural activity as well as (downward) biases in estimates of psychophysical performance.

We hypothesize that the widespread neglect to consider this effect is caused by three factors: First, there has been no generally applicable method for quantifying the effect of experimental history on perceptual decisions in psychophysical tasks. Second, there has been no systematic, direct quantification of the *magnitude* of trial-by-trial, nonstimulus determinants of perceptual decisions in the context of psychophysics with trained observers. Therefore, it is

unknown what percentage of variance in typical psychophysical data is caused by the stimulus and what percentage can be attributed to task-irrelevant experimental history. Third, although some early studies reported to find a weak or no effect of intertrial dependence on psychophysical thresholds (Senders & Sowards, 1952; Verplanck & Blough, 1958), there have been no generally applicable and practical methods for quantifying and correcting such biases on a wide range of psychophysical data. Thus, it is still an open question how strong and problematic serial dependencies are across typical psychophysical tasks.

Our focus here is on a unified statistical description of the different types of history effects in binary responses in single-stimulus or two-alternative forced-choice (2AFC) designs. Using a range of different psychophysical data sets from these paradigms, we will show that our method finds systematic dependencies between trials. Yet typical measures of psychophysical performance, such as the psychometric function’s slope and threshold, are hardly influenced by these dependencies. By performing mathematical analysis of a simplified model, we describe why classical psychometric measures are robust to sequential dependencies.

Although the kind of data considered here (binary responses) is common in psychophysics and widely used to measure perceptual thresholds with psychometric functions (e.g., Wichmann & Hill, 2001), there are also alternative psychophysical paradigms (e.g., those based on ternary responses or rating tasks) that are not compatible with this modeling framework. We do not address the question of sequential dependence in reaction times in visual search tasks, which have been characterized, e.g., by Maljkovic and Martini (2005) and Martini (2010). We also exclusively investigate intertrial sequential dependencies in behavioral tasks in which the experimenter assumes there to be none. This is in contrast to experiments investigating other internal factors violating the assumption of independent trials, such as adaptation (Chopin & Mamassian, 2012) or learning (Sugrue, Corrado, & Newsome, 2004; Corrado, Sugrue, Seung, & Newsome, 2005; Lau & Glimcher, 2005; O’Doherty, Hampton, & Kim, 2007). In such settings, the experimenter is explicitly interested in behavioral changes over time and therefore aims to induce and characterize these changes.

Methods

A statistical model for capturing serial dependencies in psychophysical data

To study the causal influence of recent experimental history on the perceptual decisions of observers, we

need a statistical model that can capture the influence of both the stimulus and previous trials on the observed response. The psychometric function is a commonly used model for psychophysical data, which usually relates the probability of a correct response to the presented stimulus intensity (Treutwein & Strasburger, 1999; Wichmann & Hill, 2001; Kuss, Jäkel, & Wichmann, 2005). To model the effect of previous trials on the response, we modify this common formulation: We relate the probability of a particular behavioral response r_t to the presented stimulus intensity \tilde{s}_t . We used “signed” stimulus intensities $\tilde{s}_t := s_t z_t$ here, which consist of the product of the absolute intensity of the stimulus s_t and an identity factor z_t , which codes when or where the target was presented. For example, in the 2AFC task (Jäkel & Wichmann, 2006) considered below, we use the stimulus identity $z_t = 1$ to indicate that the second of two presented stimuli contained a luminance increment (“target”) and set $r_t = 1$ if the observer also chose the second interval ($z_t = -1$ or $r_t = -1$ otherwise, see below and Appendix A2 for details). Choice models in psychophysics usually have a bias term δ , which captures a stimulus-independent tendency of observers to choose a particular response. To model sequential dependencies, we simply assume that δ is not constant but may shift dependent on experimental history (Treisman & Williams, 1984). This is in accordance with a large number of experimental findings (Hock, Kelso, & Schöner, 1993; Lages & Treisman, 1998, 2010; Lages & Treisman, 2010) and previous modeling attempts (Green et al., 1977; Ward, 1979; Green et al., 1980; Lockhead & King, 1983; Corrado et al., 2005; Busse et al., 2011; Bode et al., 2012; Goldfarb, Wong-Lin, Schwemmer, Leonard, & Holmes, 2012; Raviv et al., 2012). Concretely, we assume that the bias term δ can be written as a linear combination of “history features,” i.e., summary statistics of the events on preceding trials (Corrado et al., 2005; Busse et al., 2011):¹

$$\delta(\mathbf{h}_t) = \delta' + \sum_{k=1}^K \omega_k h_{kt} =: \delta' + \delta_{\text{hist}}(\mathbf{h}_t). \quad (1)$$

Here, the vector h_{kt} can be taken to be any feature of the recent history that might potentially influence behavioral responses. We say that a data set exhibits history dependence if, given the current stimulus, the current response is statistically dependent on previous stimuli and previous responses, that is,

$$P(r_t | \tilde{s}_t, \mathbf{h}_t) \neq P(r_t | \tilde{s}_t).$$

In our analyses below, we set \mathbf{h}_{kt} to be a concatenation of the last seven responses and stimulus identities, that is, $h_t = (r_{t-1}, \dots, r_{t-7}, z_{t_1}, \dots, z_{t-7})$, a vector of dimensionality $K = 14$.

The influence of history will then be modeled as a weighted sum of these history features, i.e., the history couplings ω_k in Equation 1 indicate how much the respective response/stimulus influences the current response. For example, $\omega_1 > 0$ indicates that the observer tended to repeat the previous response and $\omega_1 < 0$ that there was a tendency to switch responses. Our model captures covariations between the observer’s responses and previous responses or stimuli. These covariations could increase the variance of the resulting responses in a block² but could also lead to a decrease in variance or even leave it unchanged.

In our model, the probability of choice $r_t = 1$ (which denotes a rightward or second-interval choice in the context of 2AFC) is given by

$$P(r_t = 1 | \tilde{s}_t, \mathbf{h}_t) = \gamma + (1 - \gamma - \lambda)g \left(\delta' + \sum_{k=1}^K \omega_k h_{kt} + \alpha u_v(\tilde{s}_t) \right). \quad (2)$$

Here, $\tilde{s}_t = z_t s_t$ is the signed stimulus intensity, λ and γ describe the probabilities of stimulus-independent responses to the right (γ) or to the left (λ), and $g(x)$ is a sigmoid function. In the following, we chose the logistic function $g(x) = 1/(1 + \exp(-x))$, and δ' and α are the offset and slope of the stimulus-dependent part of the psychometric function. We note that an alternative view of the model is to assume that observers implicitly combine the stimulus with a trial-specific Bayesian prior assumption about whether the next target is in the first or second stimulus and that this prior probability $P(z_t | \mathbf{h}_t)$ depends on the recent stimulus history (Yu & Cohen, 2008; Wilder, Jones, & Mozer, 2010).

In experiments with multiple experimental conditions, we allowed the slope α to be different across conditions but assumed that the history couplings ω_k were constant across conditions. As our model describes the probability of particular responses (not the probability of the response being correct), we need to introduce a sensory threshold v , which accounts for the fact that observers perform at chance level whenever the stimulus has an intensity less than some sensory threshold v . We use the nonlinear threshold function u_v , which maps all stimuli with an intensity of less than v to zero (for details see Appendix A2 and Figure A1).

This model also has a modified intercept term δ' that can capture potential within-trial biases that are unrelated to the experimental history (see, for example, Ulrich & Vorberg, 2009, and Garcia-Perez & Alcalá-Quintana, 2011, for more detailed treatments of these effects). In some cases, these within-trial biases are also associated with differences in the observers’ sensitivity. In that case, it would be possible to apply the model separately to trials of each presentation order, which is equivalent to the approach advocated in Garcia-Perez

and Alcalá-Quintana (2011). Our formulation could also be used to include additional covariates in \mathbf{h}_t , which describe the current trial and which, thus, could be used to capture more complex within-trial effects, but this is not pursued here.

We also note that alternative parameterizations could be used to model the effect of history on the *slope* of the psychometric function. To model effects on slope, one could include features that are proportional to s_t . For example, by including a feature of the form $s_t z_{t-1}$, one could make the slope dependent on the position of the target in the previous trial.

It has been suggested that random guesses made by “undecided” observers on difficult trials in forced-choice tasks could be a source of bias in psychophysical data (García-Pérez & Alcalá-Quintana, 2010). We did not explicitly model such indecision responses. However, if observers did make guesses that are temporally correlated, this could be captured by our history features whereas a bias resulting from temporally uncorrelated guesses would affect our intercept term.

All parameters of the model were estimated from data using log-likelihood maximization (see below and Appendix A3 for details). In synthetic data sets (which satisfied our modeling assumptions), the model correctly identified the presence or absence and the magnitude of history dependence (see Appendix, Figures A2 through A5).

Parametrization of model and model-fitting

We modeled the influence of the previous seven responses and stimulus identities. To avoid having to fit 14 parameters, we only considered history features that could be described by a linear filtering process

$$h_{kt} = \sum_{t'=1}^7 b_{kt'} y_{t-t'},$$

where $y \in \{z, r\}$ and the $b_{kt'} = \eta_k^{t'-1}$ denote three exponentially decaying filter kernels with decay constants $\eta_k \in \{0, 1/2, 1/4\}$, which are sensitive to fluctuations at different time scales. Each of these filters was applied to the response sequence and to the stimulus sequence. As these basis functions are strongly correlated, they would result in strongly correlated history features, which can result in numerical problems when trying to identify their parameters from data. We, therefore, orthogonalized the basis functions with respect to each other, i.e., we ensured that they are of unit-norm and mutually orthogonal (Paninski, Pillow, & Simoncelli, 2004), resulting in new basis functions b'_k . Our 14-dimensional history feature \mathbf{h}_t thus lives in a six-dimensional subspace with its first three components given by the

projections of the previous stimulus identities onto the basis functions and the last three components given by the projections of the previous responses. Although our algorithm identifies the coefficients of these basis functions, we report the effective history filters, which can be reconstructed by multiplying the basis functions with their matching coefficients. We find the parameters α , δ' , γ , λ , the sensory threshold ν , and the history weights ω by maximizing the log-likelihood of the data under the response probabilities predicted by the model $L = \sum_t \log P(r_t | \tilde{s}_t, \mathbf{h}_t)$. This likelihood can have multiple local maxima, and there are multiple constraints on parameters (e.g., $0 \leq \lambda \leq 1 - \gamma$), which renders naive gradient-based approaches problematic. We, therefore, used the expectation maximization algorithm (Bishop, 2006), an iterative algorithm that is guaranteed to find a (local) optimum for mixture models (see Appendix A3 for details). Although this algorithm cannot guarantee convergence to a global optimum, we have found empirically that, by using a modest number of restarts combined with heuristic starting values, the algorithm typically converged to parameters that explained our data well and were close to the true parameters on simulations with known ground truth.

Performance measures, correcting for history effects and statistical tests

Performance of the models was quantified using the log-likelihood of the data under the model $L = \sum_t \log(P(r_t | h_t, \tilde{s}_t))$ across all trials indexed by t . In each trial, the choice is influenced by the effect of the stimulus $\delta_{\text{stim}}(s_t) = a \cdot u_\nu(\tilde{s}_t)$ as well as the effect of the history, $\delta_{\text{hist}}(h_t)$ (Equation 2). We thus quantified the relative influence of the history as the ratio between the variance of the history influence and the sum of the variances,

$$\text{HistCont} = \frac{\text{Var}_t(\delta_{\text{hist}}(\mathbf{h}_t))}{\text{Var}_t(\delta_{\text{hist}}(\mathbf{h}_t)) + \text{Var}_t(\delta_{\text{stim}}(\tilde{s}_t))} 100\%,$$

where Var_t indicates that the variance is determined across all trials t . This measure quantifies to what extent fluctuations in the internal decision variable can be attributed to history, and we, therefore, refer to this measure as “history contribution to variance in the decision variable.” This measure quantifies the *relative* contribution of the stimulus and the experimental history to fluctuations of the decision variable but does not model additional noise in the decision process. Thus, if the stimulus intensity is 0, the internal variance of \tilde{s} will be zero as well, and HistCont will be 100%. We excluded blocks with performance <55%.

To provide an *absolute* measure of the influence of history dependence on behavioral choice, we computed the accuracy of different models in predicting behavior: In every trial t , the model provides two probabilities, $P(r_t = 1)$ and $P(r_t = -1)$. In order to quantify how well a model predicted behavioral choice, we said that the model predicted $r_t = 1$ whenever $P(r_t = 1) > P(r_t = -1)$, i.e., in which $P(r_t = 1) > 0.5$, and said that the model predicted $r_t = -1$ otherwise. The percentage of correct predictions was calculated by counting the number of correct predictions. For the “history only” model, the parameter α was set to zero. To correct the psychometric function for the effect of errors attributable to serial dependencies, we first fitted the full model (Equation 2) to data and then extracted a psychometric function by setting the history couplings ω to zero. This model was then compared to a conventional psychometric function (Equation 1). Confidence intervals for history kernels were determined using a bootstrap procedure. After the values of the history features in each trial had been calculated, 2,000 bootstrap data sets of the same size were sampled from the data with replacement (Efron & Tibshirani, 1993). Confidence intervals were defined as the 2.5 and 97.5 percentiles of this distribution. As the full model has six more parameters than the history-free model, bootstrap samples cannot be used to evaluate the statistical significance of history features (as, on each bootstrap set, the full model would have a higher likelihood than the history-free one). We therefore used a permutation test: We permuted the sequence of trials randomly such that the history features could contain no information about the response but that the association between stimulus and response was left intact. Thus, the permuted data sets yielded an approximation of the distribution of likelihoods that would be expected under the null hypothesis of no serial dependence. We fit the model to 2,000 permutations of the original data sets and compared performance measures and other parameters to the 95th percentile of this permutation distribution. In cases in which the effect of history dependence is nonlinear, our model will capture a linear approximation to this nonlinear system and will detect the presence of history dependence provided that its linear term is not negligible. Of course, weak history dependence on small data sets might not reach the level of statistical significance and might therefore not be detected by the statistical tests described here.

Details of psychophysical data

All psychophysical experiments analyzed in this study were conducted in accordance with the regula-

tions of the relevant institution (Max Planck Institute Tübingen and TU Berlin). We analyzed data from four different experiments in which one of the coauthors was actively involved (FAW); all data were considered “clean” by conventional analyses, i.e., showed no obvious signs of learning, fatigue, or equipment failure as determined by criteria outlined elsewhere (Wichmann & Hill, 2001).

Five observers had to detect luminance increments in different fields of Adelson’s checkerboard illusion (Maertens & Wichmann, 2012) (for a description of the paradigm, see Maertens & Wichmann, 2013). Each observer performed between 2,448 and 3,204 temporal 2AFC trials and received only overall feedback after blocks of 36 trials, i.e., the average performance over 36 trials. Each 36-trial block contained 18 trials for each of the 2AFC stimulus orderings in a randomly shuffled trial order. As our permutation test applied to this data would also have a fixed number of alternatives in the null distribution, it will also work correctly if blocks are constructed by shuffling rather than by sampling from a binomial distribution. In total, 14,256 trials from this experiment were analyzed.

Five observers participated in a temporal 2AFC plaid masking experiment (Wiebel & Wichmann, 2007); observers received auditory trial-by-trial feedback about whether or not their responses were correct. After practice sessions, the observers performed between 3,255 and 4,100 trials. In these trials, the signal stimulus was randomly assigned to one of the two intervals, independently of all other trials. In total, 18,305 trials from this experiment were analyzed.

Six observers performed a single-interval (yes-no) auditory tone noise-detection experiment. Each observer performed between 25 to 33 sessions. The initial seven to 12 sessions were used for training, and only the remaining 18 to 22 sessions were analyzed. Data were collected in blocks of 50 to 60 trials. Whether a particular trial contained a signal or not was independent of all other trials. Observers received single-trial feedback for the first 10 trials of each block, and we, therefore, discarded these trials. For each observer, between 20,610 and 22,800 trials were analyzed resulting in a total of 127,430 trials (Schönfelder & Wichmann, 2013).

Finally, we analyzed data from a study by Jäkel and Wichmann (2006). From this study, we analyzed data from six observers, five naive at the beginning of the experiment and one highly trained observer. For each observer, we analyzed between 2,640 and 5,665 spatial 2AFC trials, i.e., a total of 27,060 trials. The signal was randomly presented either left or right of the fixation cross. The assignment of the signal to the left or right position did not depend on signal positions in previous trials.

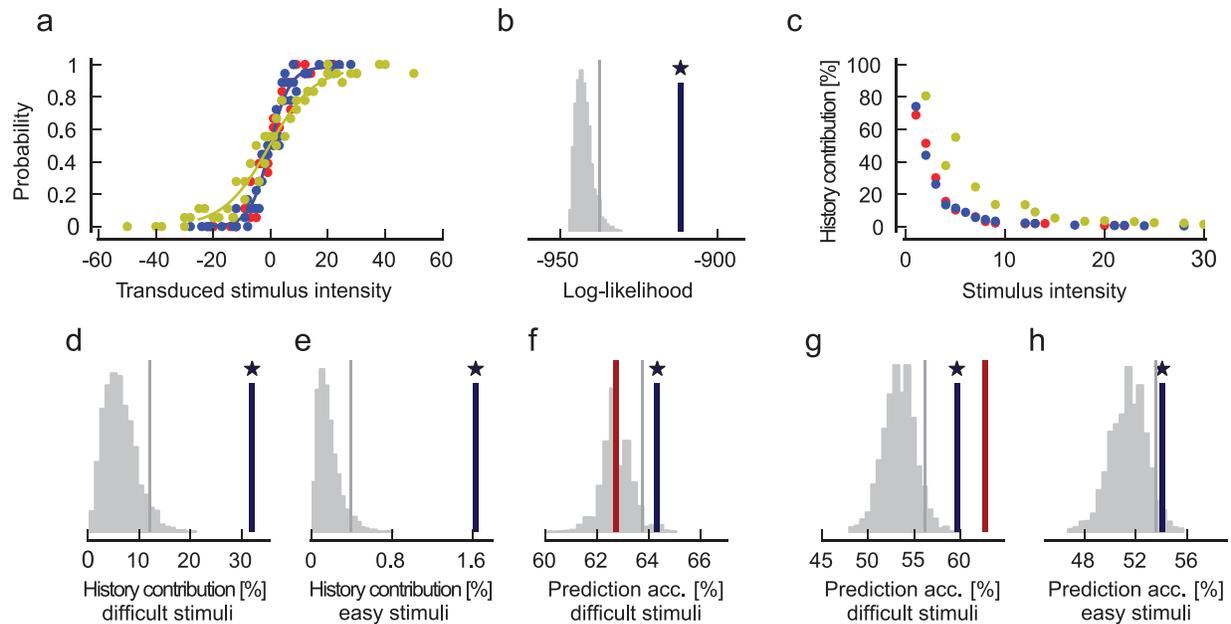


Figure 1. Data from example observer in luminance experiment. (a) Psychometric function and proportion of “second interval” responses as a function of transduced stimulus intensity for observer pk; colors correspond to different experimental conditions. (b) Log-likelihood of the full model (dark blue line). Here and in all panels, the gray histogram is the distribution on permuted data (gray), and the vertical gray line marks its 95th percentile; the star marks statistical significance. (c) History contribution to variance in the decision variable as a function of stimulus intensity. (d) History contribution to variance in the decision variable by history on difficult trials. (e) Same as (d) but for easy trials. (f) Prediction performance (percentage correct) of full model (including stimulus and history terms, dark blue line) in predicting observers’ responses on difficult stimuli and comparison with stimulus-only model (red line). (g) Prediction performance of model with only history dependence and no stimulus dependence (dark blue line) and comparison with stimulus-only model (red line). (h) Prediction performance of history-only model on easy stimuli (dark blue line).

Results

History dependence in low-level visual psychophysical task

First, we show an example of a human observer whose decisions are influenced by the recent experimental history. We analyzed data from a two-interval forced-choice experiment in which a human observer had to decide whether a luminance increment appeared in the first or the second of two temporally separated presentations of Adelson’s checkerboard (Maertens & Wichmann, 2012) (see Methods for details). Performance in the task showed a clear and clean dependence on stimulus intensity (Figure 1a) as intended by the experimenter. Nevertheless, our model, which accounts for history dependence, provided a much better explanation of the data than a conventional model that assumes independent trials: The log-likelihood of our model ($l_{\text{full}} = -911.7$) was substantially larger than the one of the independent-trial model ($l_0 = -941.2$, corrected for difference in parameters using Akaike’s information criterion, AIC), and a permutation test against random trial sequences revealed that this

performance benefit was significant at level $p < 0.0005$ (Figure 1b, see section “Performance measures, correcting for history effects and statistical tests” for details; all p values in the following are derived from this test unless stated otherwise).

To show that the effect of previous trials was not only statistically significant, but determined behavior noticeably, we calculated how much of the trial-by-trial variability in the observer’s decision variable could be explained by history (see Methods for details). This quantity (history contribution to variance in the decision variable) would be 0% if trials were independent and 100% if observers were exclusively influenced by the experimental history. (It would also be trivially be 100% in tasks or conditions in which there is no signal.) The influence of history decreases monotonically with stimulus intensity (Figure 1c). On difficult stimuli (for which the performance of the observer was between 55% and 75% correct), 32% of the variance of the observer’s decision variable depended on previous trials rather than the current stimulus ($p < 0.0005$, Figure 1d). As expected, behavior was primarily explained by the stimulus on easy trials (for which performance $> 75\%$ correct), on which 98% of the variance of the observer’s decision variable was explained by the stimulus, and only 1.6% of the

variance of the observer's decision variable was still history-dependent ($p < 0.0005$, Figure 1e).

To elucidate the ability of the stimulus and experimental history to predict behavior, we quantified in how many trials the different models correctly predicted the subjects' decision: The prediction performance of the history-only model (i.e., for which the stimulus weight is set to zero) decreased with stimulus intensity (Appendix A6). For difficult stimuli, the full model (i.e., stimulus and history) predicted behavior with 64% accuracy (Figure 1f) whereas the stimulus only achieved 63% and the history alone achieved 60% ($p < 0.0005$, chance level is 50%, Figure 1g). Our analysis shows that, on difficult stimuli, which are of relevance for measuring psychophysical performance, perceptual decisions of this observer are almost as strongly influenced by internal decision variable fluctuations induced by experimental history as they are by the experimental stimulus. Nevertheless, the fact that the prediction power was far away from 100% implies that much of the variability in subjects' responses could not be accounted for by the stimulus or experimental history.

Even on easy stimuli, prediction performance of the history-only model was statistically significant at 54% ($p < 0.03$, Figure 1h).

The additional benefit of the “full” model over the stimulus-only model might seem surprisingly small. This might be interpreted as indicating that the stimulus and the history signal were very correlated. However, this was not the case: For this observer, less than 0.05% of the stimulus component could be explained from the history component (maximum across observers was 0.11%). To better understand this effect and to assess whether our results about history contribution to variance in the decision variable were consistent with these measures of prediction performance, we investigated a simplified model in which the stimulus, the history influence, and the internal noise were approximated by independent Gaussian random variables. The variances of stimulus and history were chosen to be consistent with our results on history contribution to variance in the decision variable, the variance of the internal noise distribution was adapted to match the observers' performance in difficult trials. In this model, the predictive power of the history-only model was 59%, of the stimulus-only model 63%, and of the combined model 65%, which is in agreement with our empirical findings. On average across observers, this simplified model explained prediction accuracies with an error of 3%. Therefore, the small gain of the combined model relative to the other two models is consistent with our other findings and, in particular, does not imply that the history is correlated with the current stimulus.

Effect of history dependence on psychometric functions

If observers are influenced by task-irrelevant information in previous trials, then this could lead to suboptimal performance in the task. Hence, estimates of the capabilities of sensory processing systems could be negatively biased by the dependence of perceptual decisions on internal states. Our model allows us to provide a statistical description of psychophysical data that separates the effects of experimental history and the stimulus on the decision, and thus, it leads to a “decontaminated” estimate of the psychometric function. We emphasize that this “discounting” of history-induced errors yields a model-based estimate of psychophysical performance, which could be wrong if the modeling assumptions were inappropriate. Our model gives two generally different psychometric functions for the two response intervals (Ulrich & Vorberg, 2009; Garcia-Perez & Alcalá-Quintana, 2011). We report the average of these two psychometric functions here because both psychometric functions were typically similar, and our interest is on trial-by-trial effects rather than biases that might emerge within an individual trial.

After discounting errors induced by history, the predicted probabilities of correct responses differed from a conventional psychometric function (Figure 2a and 2b), i.e., some of the observers' errors could be attributed to the influence of task-irrelevant features. We quantified the impact of the history on the psychometric function by comparing the stimulus level at which our model or the independent trial model predicted a performance of 85% correct. Accounting for history dependence yielded discrimination thresholds that were reduced to 96% of the original value, i.e., a reduction of, at most, 4% ($p < 0.0005$, average across conditions 97.2%, Figure 2c). Thus, the fact that conventional analyses cannot discount the errors induced by dependence on previous trials leads to a slight underestimation of the stimulus sensitivity of this observer.

On the one hand, experimental history clearly influences behavior, but on the other hand, it only seems to have a weak effect on psychophysical thresholds. To better understand this apparent paradox, we considered a second simplified, analytically tractable model (see Appendix A6 for details). We note that, if the psychometric function were perfectly linear, even strong history dependence would not lead to *any* degradation in performance as stimulus-independent lapses in one direction and in the other direction would cancel perfectly. However, the psychometric function is a nonlinear model. We found that a history-induced *standard deviation* of $\sigma = \text{Std}(\delta_{\text{hist}}(h_i))$ leads to a quadratic reduction in the slope of the psychometric

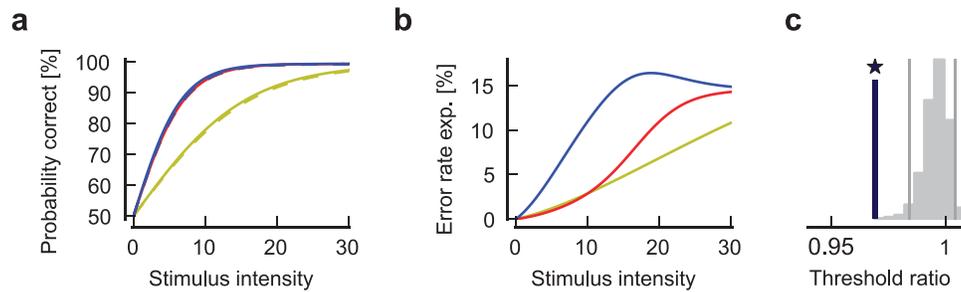


Figure 2. Effect of history dependence on the psychometric function for the example observer. (a) Psychometric functions indicating frequency of correct responses as a function of stimulus intensity. Dashed lines mark fits of a model without history terms. Colors correspond to different experimental conditions. (b) Percentage of behavioral errors attributable to history, i.e., normalized difference between error rates predicted psychometric functions with or without history couplings. (c) Ratio of 85% performance thresholds (blue line) between full model and conventional model and null distribution (gray histogram). Thresholds are lower for the full model with history terms.

function, i.e., one that is proportional to the history-induced variance σ^2 . Thus, for typical history dependence ($\sigma < 1$), the reduction in the slope is even smaller (as $\sigma^2 < \sigma$). Furthermore, the proportionality factor of this relationship is also small, namely $\pi/16 \approx 0.2$. For example, for a standard deviation of $\sigma = 0.3$, one would only expect a reduction of the slope by $0.09 \times \pi/16 \approx 1.18\%$.

This relationship thus explains why even substantial history dependence only has a weak effect on the slope of the psychometric function (and hence thresholds) (Verplanck & Blough, 1958). Conversely, including history dependence in the model should lead to a steeper psychometric function and, thus, lower thresholds. For our example observer (for which $\sigma = 0.52$), this simplified analysis predicted a reduction of threshold by 5.3%, which slightly overestimates the empirically measured reduction of 4%. Across observers, the threshold changes predicted by this simplified model were correlated with those of the full model ($c = 0.73$) and did not differ in their mean level (paired samples t test: $t(21) = 0.57$, $p = 0.57$).

Serial dependencies across multiple observers and paradigms

To demonstrate that the results above are not an idiosyncrasy of this particular observer or task, we analyzed data from 22 human observers collected in four different experimental paradigms, resulting in a substantial data set of 187,051 trials in total. In all cases, observers were engaged in low-level psychophysical experiments, and the data were considered “clean” by means of conventional analyses (Wichmann & Hill, 2001). However, stimulus material and task varied widely across the most commonly used experimental paradigms, including visual 2AFC tasks with the alternatives separated spatially and temporally and

an auditory single-interval task. In experiments with multiple experimental conditions, we conservatively assumed the structure of history dependence to be fixed across conditions. Thus, if observers have condition-dependent history dependence, our results would underestimate the true magnitude of history dependence.

We found significant history-dependence ($p < 0.05$) in 19 out of 22 observers, and modeling the non-stimulus determinants of the behavioral choices led to an average increase in log-likelihood of 0.009 ± 0.0020 per trial (*SEM* across observers, Figure 3a, see Appendix A7 through A12 for detailed results of two further observers). In other words, a data set of 500 trials would be (on average) 77 times more likely under our model than under a conventional model assuming independent trials. Significant history dependence was found in all four experimental paradigms we investigated (see Figure 3a), and its strength varied considerably across observers.

On average, $13.7\% \pm 2.4\%$ (*SEM*) of variance of the decision variable on difficult stimuli was determined by the experimental history (Figure 3b) and not by the presented stimulus with values for individual observers as high as 48.2%. Experimental history was a meaningful predictor of behavioral choices in individual trials. On average, the model based on previous trials predicted $56.5\% \pm 1.0\%$ (*SEM*, chance level 50%, significant for 16 out of 22 observers) of responses on difficult stimuli correctly, compared to $64.8\% \pm 0.5\%$ for the stimulus and $65.5\% \pm 0.4\%$ for the combined model (Figure 3c). Across observers, the prediction performance of the full model was significantly better than for a model that only contained stimulus terms ($p < 10^{-3}$, permutation test of paired differences). For one observer, the history was, in fact, a better predictor of the behavioral choice than the presented stimulus.

We emphasize that the prediction accuracy of the history model was on par with the accuracy levels

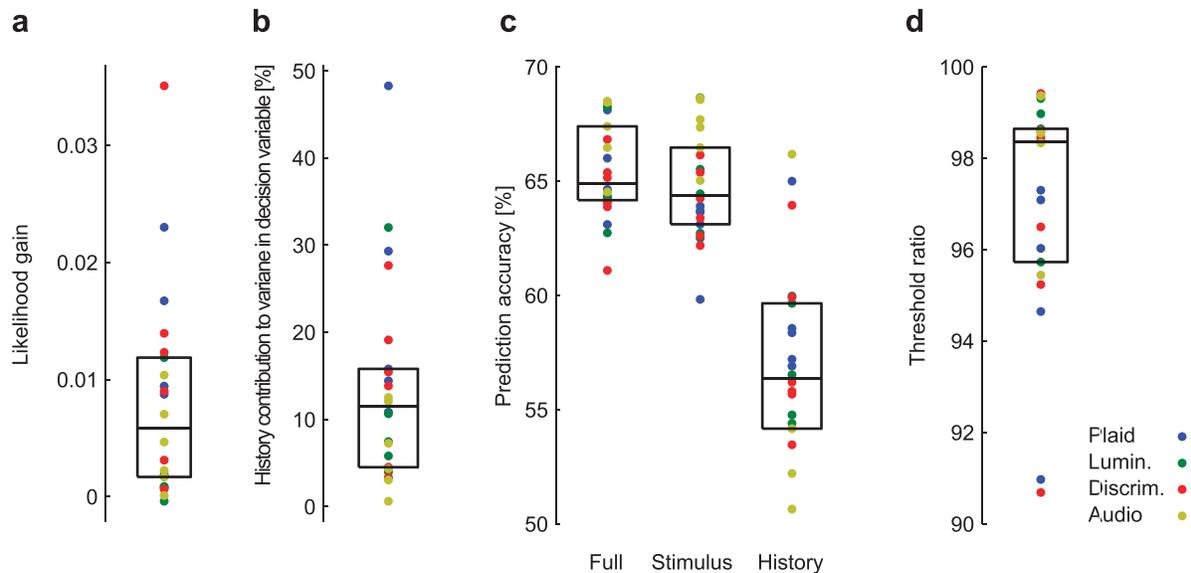


Figure 3. Summary results across observers. (a) Likelihood change due to history: Colored dots mark the increase in likelihood with respect to a model with no history terms, corrected for different numbers of parameters using Akaike's criterion. Black box marks median and quartiles of distribution. (b) History contribution to variance in the decision variable on difficult trials: Colored dots mark the fraction of total history contribution to variance in the decision variable on difficult trials. (c) Prediction of behavioral responses from full model using both history and stimulus (left), the stimulus only (center), or the experimental history only (right). (d) Ratio between 85% performance thresholds estimated with the full model and with a model without history terms. Thresholds were consistently overestimated if history dependence was not taken into account.

reported in many decoding studies, which predict behavioral choice from functional imaging measurements (e.g., Soon, Brass, Heinze, & Haynes, 2008); thus, although the absolute numbers seem low, they could potentially be big enough to lead to confounds in the analysis of imaging data (Lages & Jaworska, 2012). As expected, performance on easy stimuli was largely driven by the stimulus, and previous trials explained only $1.4\% \pm 0.3\%$ of the variance of the decision variable and predicted choices at $52.7\% \pm 0.5\%$ correct. We also note that the variance across observers is substantial, i.e., that there are large interpersonal differences in how strongly observers are affected by experimental history.

Across observers, the thresholds that were estimated by the model with history terms were $97.0\% \pm 0.5\%$ of those obtained without history (significant for 17 out of 22 observers). In other words, the observers' sensitivity was, on average, 3.0% higher if errors caused by a reliance on task-irrelevant information in previous trials were discounted. Thus, history dependence leads to a small yet systematic underestimation of perceptual sensitivity. Nevertheless, in most analysis questions, such small differences in estimated thresholds will be negligible. The upper asymptotic performance was not systematically closer to 100% if history was modeled (reduction in asymptotic error rate by $0.2\% \pm 0.2\%$).

Finally, we also tested our modeling assumption that history dependence would primarily lead to *shifts* in the

psychometric functions and not changes in the slope. We fit four conventional psychometric functions separately to trials that followed a left response and trials that followed a right response as well as trials that followed a left stimulus and trials that followed a right stimulus. Differences in horizontal shifts were, on average, 2.3 times larger than differences in slopes (after normalizing with the estimates' standard errors). In 35 out of 44 cases, the effect of the previous trial on the horizontal position of the psychometric function was larger than the effect on the slope of the psychometric function ($p = 0.0001$ binomial test).

What properties of the recent experimental history were predictive of behavior? To answer this question, we visualized the filtering kernels, which capture how previous responses and previous stimulus identities (i.e., target locations) influence the decision in the current trial. The exemplary observer from the luminance experiment was mostly sensitive to responses in the most recent trial and showed a tendency to avoid previous choices as evidenced by the predominantly negative filter weights (Figure 4a). In contrast, the previous stimulus identities did not have a significant effect on decisions. This finding is in accordance with the fact that this observer did not receive trial-by-trial feedback about the identity of the preceding stimulus. Figure 4b displays response kernels for all observers and shows that the strongest influence was from the most recent trial (55.6% of the history contribution to

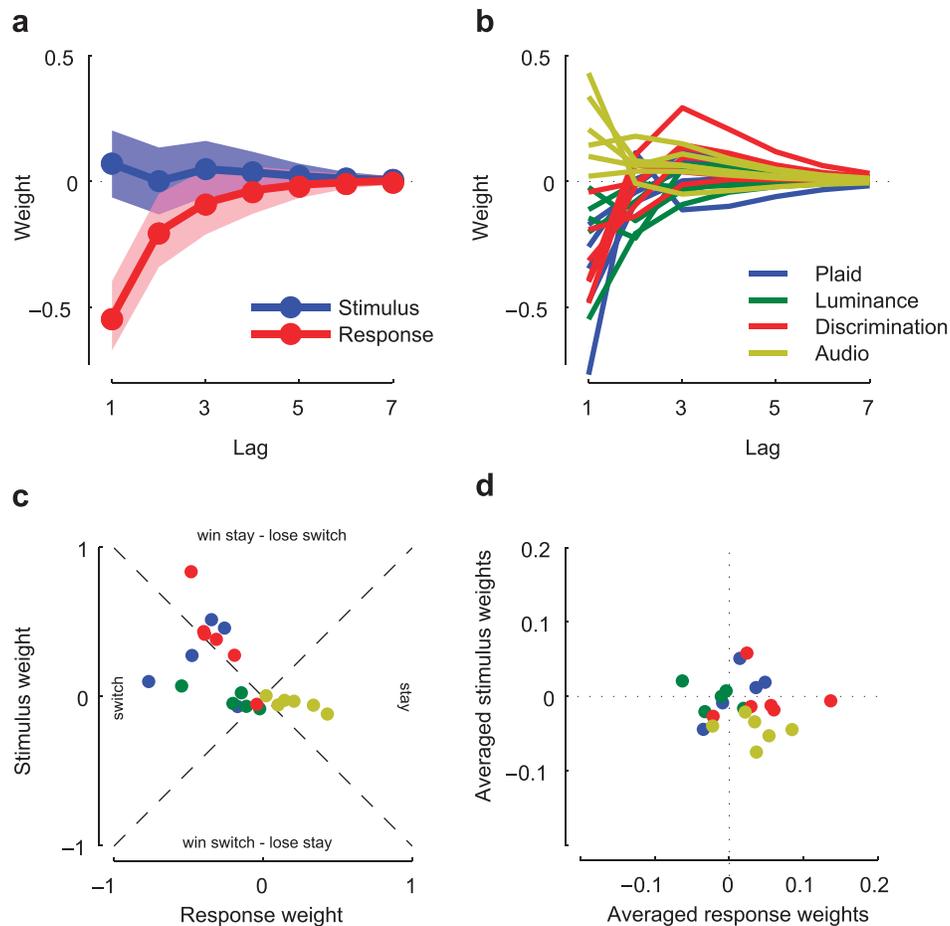


Figure 4. History couplings. (a) Weights assigned to previous stimuli and responses for observer pk. Shaded regions mark 95% bootstrap confidence regions. (b) Response kernels for all observers. Color codes for the different experiments. (c) Each position in this coordinate system corresponds to a behavioral strategy. For example, a measurement being in the upper quadrant shows that the observers had a tendency to repeat a response after a successful trial and to alternate otherwise. Weights assigned to previous response and previous stimulus identity across observers. (d) Average of weights assigned to stimuli and responses more than one trial back.

variance in the decision variable was explained by the previous trial; all kernels are shown in Appendix A8). Nevertheless, there was a significant influence of longer trial lags, which contributed a log-likelihood gain of 0.0013 ± 0.004 (*SEM*) per trial across all data sets (corrected using AIC, $p = 0.00001$ binomial test).

The combination of the filter weights associated with the previous stimulus or response can be viewed as the behavioral strategies employed by different observers (Figure 4c). Negative filter weights associated with the previous response indicated a tendency to switch, and negative filter weights for the previous stimulus identity indicated a tendency to avoid the previous target. The top half of the plot contains a continuum of “win stay–lose switch” strategies, i.e., it signifies a tendency to repeat a response if it was successful and to otherwise alternate the response. Interactions between these effects also give rise to more interesting strategies: For example, along the upper part of the negative diagonal,

the weights for previous stimulus and the previous response have the same magnitude but opposite sign—thus, these effects cancel after correct responses (on which the previous stimulus equals the previous response), but they lead to history effects after incorrect responses. Observers tend to switch their responses after errors, a strategy that could be called “lose-switch.” A similar reasoning can be applied to the other diagonals as well.

The observers in the luminance experiment consistently associated negative weights to previous responses (average -1.13 ± 0.38 , *SEM*), reflecting a tendency to switch responses from trial to trial. Weights associated with previous stimuli were heterogeneous and, on average, a factor of 3.85 ± 1.98 times smaller than the response weights. In contrast, most observers from two masking experiments (plaid mask in the first experiment and a sine grating mask of the same spatial frequency and orientation in the second experiment)

are near the upper left diagonal of this plot, reflecting the fact that these observers primarily changed their response criterion after errors, which is likely to be a consequence of the fact that they received trial-by-trial feedback. In contrast, subjects in a yes-no audio experiment had weak weights associated with the previous stimulus (average weight -0.11) and stronger weights associated with previous responses (average weight 0.33). Thus, these observers showed slow fluctuations of their decision variable, which manifests itself in a tendency to repeat their previous responses. Although the overall influence for longer trial lags was substantially weaker, a clear clustering of weights according to experimental paradigm was still evident (Figure 4d).

We note that these filtering kernels are not equivalent to cross-correlations between current and previous responses. For example, a direct effect that is confined to the previous trial would lead to cross-correlations even at (in theory) infinite time lag with an exponentially decaying strength. Similarly, and in contrast to correlations, these filter kernels allow us to quantify whether the response of the subject is dependent on previous responses or previous stimulus identities. For example, for our example observer, the cross-correlation between previous stimulus identities and the current response is nonzero (as previous stimulus identities are correlated with previous responses correlated with the current response) despite the fact that our analysis reveals that their *direct* influence is negligible. Thus, with our method, we can disentangle serial dependence on previous responses from a dependence on previous stimuli or feedback.

Discussion

Our results demonstrate that there exist significant and systematic causal determinants of perceptual decisions over and above the stimulus presented to the organism. Great care must thus be exercised when attempting to infer mechanisms from behavioral data unless the unwanted, non-stimulus-dependent internal factors are taken into consideration. In particular, we showed that the predictive power of experimental history was similar to values reported in studies predicting behavioral choices from functional imaging measurements (Soon et al., 2008; Lages & Jaworska, 2012). After correcting for the effect of history dependence, observers were slightly more sensitive to the stimulus (i.e., had lower thresholds) than determined by conventional analyses. Our experiments were conducted with experienced observers and in tightly controlled settings. In addition, our model assumes the effect of history to be linear. If the true history

dependence in the data is nonlinear, our model will only capture the linear kernel of the nonlinear system. It is therefore likely that we have explored the lower bound of serial dependence in behavioral studies. In addition, these effects might well be more pronounced for untrained observers, patients, animals, or in other experimental designs, e.g., those employing adaptive procedures (Treutwein, 1995). Therefore, it might be unwarranted to extrapolate our findings or previous reports to other experimental settings.

The existence of serial dependencies has long been known (Verplanck et al., 1952; Senders & Sowards, 1952; Howarth & Bulmer, 1956). Although these dependencies have a modest influence on aggregated measures of performance (see also Senders & Sowards, 1952; Verplanck & Blough, 1958), trial-by-trial effects are consistent in individual trials. Our results show that, even for low-level perceptual tasks, perceptual decision-making cannot be modeled as being based on a feed-forward system that passively responds to external stimuli. As observers were strongly influenced by their previous responses—and not by previous stimuli—adaptation is unlikely to be an explanation for the observed intertrial dependence found here: If the response patterns could be explained by adaptation, we would expect observers to avoid the response associated with previous stimuli. This should result in negative history weights associated with previous stimuli. Yet, for most observers, history weights associated with previous stimuli are very close to zero. Similarly, the fact that history dependence was also observed in experiments without trial-by-trial feedback makes it unlikely that either posterror dynamics (Goldfarb et al., 2012) or models of reinforcement learning (Dayan & Niv, 2008) could explain our results. In contrast, a more likely explanation for our results would be that human observers combine information from the stimulus with their prior beliefs about where they expect the next target. If they have an incorrect model of what constitutes a random sequence of events (Bar-Hillel & Wagenaar, 1991) and combine their single-trial expectations with stimulus information in a Bayesian fashion (Körding & Wolpert, 2004), then this would lead to precisely the type of sequential dependencies observed here.

Neglecting sequential dependencies can be problematic for four reasons: First, as we showed above, intertrial dependencies can lead to small but systematic biases in characterizations of the performance limits of a sensory system. In addition, these biases might be bigger for behaving animals or other experimental paradigms. Explicitly analyzing sequential dependencies can help in identifying these biases if they occur. Second, by analyzing dependencies across trials, researchers might obtain interesting information from psychophysical data. For example, Chopin and Ma-

massian (2012) analyzed sequential dependencies to draw conclusions about adaptive processing of orientation, which would be inaccessible with averaged data. Third, statistical techniques for system identification have gained popularity in the perceptual sciences over the last years (Ahumada & Lovell, 1971; Kienzle, Franz, Schölkopf, & Wichmann, 2009; Macke & Wichmann, 2010; Schönfelder & Wichmann, 2012). Therefore, it is conceivable that the application of system identification techniques to data contaminated by serial dependencies may recover wrong features or may show a disappointingly low correlation between inferred features and behavior. Fourth—and arguably most importantly—many studies relating behavioral choices to measurements of neural activity or their correlates are dependent on the assumption that the perceptual choice is determined by the current stimulus (Fründ, Busch, Schadow, Körner, & Herrmann, 2007; Soon et al., 2008; Busch, Dubois, & VanRullen, 2009; Nienborg & Cumming, 2009; Tong & Pratte, 2012) and could be led astray if internal states are a strong determinant of these choices. Similarly, the fact that behavioral choices can be predicted from neural measurements could be a consequence of sequential dependencies combined with a dependence of neural activity on previous choices (Lages & Jaworska, 2012).

Previous studies have reported history kernels for human reaction times (Maljkovic & Nakayama, 1994; Maljkovic & Martini, 2005; Martini, 2010), binary responses in animals (Corrado et al., 2005; Lau & Glimcher, 2005; Busse et al., 2011), and human categorization judgments (Stewart et al., 2005) that share qualitative similarities with the ones presented here. Furthermore, if effects in earlier studies had been expressed as kernels, these might also have supported this perspective (Senders & Sowards, 1952; Treisman & Williams, 1984). Nevertheless, given that these kernels were estimated in very different experiments or even using different measures, they are likely to be caused by different underlying mechanisms. Kernels estimated in studies of reaction times typically indicate that repetition of the same response becomes faster (Maljkovic & Nakayama, 1994; Maljkovic & Martini, 2005; Martini, 2010). Kernels derived describing an animal's reward expectation reveal a tendency to expect reward in the same position (Corrado et al., 2005; Lau & Glimcher, 2005) or a combination of reward expectancy and sensory processes (Busse et al., 2011). In contrast, we note that studies of sequential dependence in “purely” perceptual processing—with two stimuli that are themselves difficult to discriminate—have typically not derived response kernels (Senders & Sowards, 1952; Verplanck et al., 1952). In addition, the classical studies of these effects (Senders & Sowards, 1952; Verplanck et al., 1952) used long sequences of stimuli with the same intensity and no

intervening blank stimuli in single-interval designs, a design that has been rarely used since the introduction of signal-detection theory (Green & Swets, 1966). Some authors have explained sequential dependencies by means of a temporally varying criterion (Treisman & Williams, 1984; Lages & Treisman, 1998). We find consistent sequential dependencies in many different experimental designs, including forced-choice designs, and these dependencies have a systematic—albeit small—effect on performance. Although sequential dependencies in different experiments have very different meanings and mechanisms, the statistical framework that we present can describe all of the history effects that refer to binary responses. Thus, our framework allows for a unified quantification and comparison of history effects even if they have very different psychological interpretations.

Previous studies typically found that the experimental history mainly shifted psychometric functions horizontally (Hock et al., 1993; Lages & Treisman, 1998, 2010; Lages & Treisman, 2010). Therefore, our model resembles previous models that also described sequential dependencies as a form of trial-by-trial response bias rather than variations in an observer's sensitivity to the stimulus (Green et al., 1977; Ward, 1979; Green et al., 1980; Lockhead & King, 1983; Corrado et al., 2005; Busse et al., 2011; Bode et al., 2012; Goldfarb et al., 2012; Raviv et al., 2012). (Note that we here refer to psychometric functions that relate the stimulus intensity to the probability of a given binary response and *not* to the probability of a *correct* response.) Yet it is possible that history has an influence on the slope of the psychometric function as well (see Lages & Treisman, 2010, for some indication of this). Our model is not able to capture these kinds of sequential dependencies, and thus, the history dependence reported here should be treated as a lower bound.

The statistical methodology we presented makes it possible to quantify the strength of intertrial dependence and to correct psychometric functions or other estimates of behavioral performance for the systematic influence of previous trials. Our framework makes it possible to track fluctuations of the internal decision variables on *individual* choices (Lau & Glimcher, 2005; Corrado & Doya, 2007; O'Doherty et al., 2007) and thus has the potential to reveal a rich source of information that had previously been buried by trial averaging. Given that time series of behavioral observations are ubiquitous in neuroscience and related fields, our methods will be applicable to a wide range of experimental or clinical paradigms that measure human or animal performance. Combined with methods for single-trial analyses for neurophysiological recordings (Churchland, Yu, Sahani, & Shenoy, 2007), they thus

have the potential to contribute to a more realistic understanding of perceptual decision-making.

Keywords: perceptual decisions, psychophysics, statistical modeling, serial dependence, internal variability

Supplementary material

Appendices A1 to A8 can be found in the Supplementary Information file. A software implementation of the statistical methods presented in this paper is available at https://bitbucket.org/mackelab/serial_decision.

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Footnotes

¹Letting the bias term vary dynamically from trial to trial complicates the interpretation of the model in terms of a signal and a decision criterion as defined in signal-detection theory. We here restrict ourselves to finding a statistical description of history effects, and we do not attempt to disentangle whether they influence the criterion or the signal in the decision process.

²Here, a block refers to a sequence of trials with constant (unsigned) stimulus intensity. Thus, trials within a block differ only with respect to the position of the stimulus (e.g., left/right in a spatial 2AFC) and their history features.

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